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# Cross-shore distribution of longshore sediment transport: comparison between predictive formulas and field measurements

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## Abstract

The skill of six well-known formulas developed for calculating the longshore sediment transport rate was evaluated in the present study. Formulas proposed by Bijker [Bijker, E.W., 1967. Some considerations about scales for coastal models with movable bed. Delft Hydraulics Laboratory, Publication 50, Delft, The Netherlands; Journal of the Waterways, Harbors and Coastal Engineering Division, 97 (4) (1971) 687.], Engelund–Hansen [Engelund, F., Hansen, E., 1967. A Monograph On Sediment Transport in Alluvial Streams. Teknisk Forlag, Copenhagen, Denmark], Ackers–White [Journal of Hydraulics Division, 99 (1) (1973) 2041], Bailard–Inman [Journal of Geophysical Research, 86 (C3) (1981) 2035], Van Rijn [Journal of Hydraulic Division, 110 (10) (1984) 1431; 110(11) (1984) 1613; 110(12) (1984) 1733], and Watanabe [Watanabe, A., 1992. Total rate and distribution of longshore sand transport. Proceedings of the 23rd Coastal Engineering Conference, ASCE, 2528–2541] were investigated because they are commonly employed in engineering studies to calculate the time-averaged net sediment transport rate in the surf zone. The predictive capability of these six formulas was examined by comparison to detailed, high-quality data on hydrodynamics and sediment transport from Duck, NC, collected during the DUCK85, SUPERDUCK, and SANDYDUCK field data collection projects. Measured hydrodynamics were employed as much as possible to reduce uncertainties in the calculations, and all formulas were applied with standard coefficient values without calibration to the data sets. Overall, the Van Rijn formula was found to yield the most reliable predictions over the range of swell and storm conditions covered by the field data set. The Engelund–Hansen formula worked reasonably well, although with large scatter for the storm cases, whereas the Bailard–Inman formula systematically overestimated the swell cases and underestimated the storm cases. The formulas by Watanabe and Ackers–White produced satisfactory results for most cases, although the former overestimated the transport rates for swell cases and the latter yielded considerable scatter for storm cases. Finally, the Bijker formula systematically overestimated the transport rates for all cases. It should be pointed out that the coefficient values in most of the employed formulas were based primarily on data from the laboratory or from the river environment. Thus, re-calibration of the coefficient values by reference to field data from the surf zone is expected to improve their predictive capability, although the limited amount of high-quality field data available at present makes it difficult to obtain values that would be applicable to a wide range of wave and beach conditions. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* Longshore sediment transport; Predictive formulas; Field measurements

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## 1. Introduction

During the past three decades, numerous formulas and models for computing the sediment transport by waves and currents have been proposed, ranging from quasi-steady formulas based on the traction approach of Bijker, and the energetics approach of Bagnold, to complex numerical models involving higher-order turbulence closure schemes that attempt to resolve the flow field at small scale. There are relatively few high-quality field data sets on the cross-shore distribution of the longshore sediment transport rate available to evaluate existing predictive formulas. Kraus et al. (1989) and Rosati et al. (1990) measured the longshore transport rate across the surf zone using streamer traps (i.e., DUCK85 and SUPERDUCK field experiments). Miller (1999) measured the cross-shore distribution with optical backscatter sensors (OBS) combined with current measurements (i.e., SANDY-DUCK field experiment). The measurements reported by Miller (1999) covered a number of storms, thus complementing the measurements by Kraus et al. (1989) and Rosati et al. (1990) that were made in milder swell waves.

The objective of the present study is to evaluate the predictive capability of six well-known sediment transport formulas, adapted to calculate the cross-shore distribution of the longshore sediment transport rate, based upon the above-mentioned three field data sets. We selected formulas that have gained world-wide acceptance in confidently predicting longshore sediment transport rates. This, however, should not be interpreted as a sign of disagreement or a lessening of the importance of formulas not discussed here. Only sand transport was investigated in this study, and focus is on computing the time-averaged net longshore transport rate.

Background to the investigated sediment transport formulas are given in Section 2 and their main characteristics are summarized (the equations used to calculate the transport rate are given in Appendix A). Next, in Section 3, the longshore sediment transport data sets are described. In Section 4 are shown the results of the comparisons between the formulas and the field data including a wide range of wave and current conditions. An overall discussion of the results from the comparisons is provided in Section 5, where the strength and weaknesses of the investigated formulas are assessed, as well as their limitations, using various statistical measures. Finally, the conclusions of the study are presented in Section 6.

## 2. Longshore sediment transport formulas

Longshore sand transport is typically greatest in the surf zone, where wave breaking and wave-induced currents prevail, although a pronounced peak can be found in the swash zone as well (Kraus et al., 1982). Typically the total (or gross) longshore sediment transport rate is computed with the CERC formula (SPM, 1984) in engineering applications. However, as ability to predict the surf zone hydrodynamics has improved, the need for reliable formulas that spatially better resolves the sediment transport rate has increased, both concerning the cross-shore distribution of the transport rate and the concentration distribution through the water column.

In this investigation, the skill of six published formulas proposed for calculating the cross-shore distribution of the longshore sediment transport rate was investigated. Transport rates were calculated for the utilized cases using standard coefficient values (as given in the literature) without calibration. In the present comparison the formulas proposed by Bijker (1967, 1971), Engelund and Hansen (1967), Ackers and White (1973), Bailard and Inman (1981), Van Rijn (1984), and Watanabe (1992) (as they chronologically appeared in the literature) were employed, representing the most common approaches for calculating the time-averaged net sediment transport rate. The Bijker and Van Rijn formulas also calculate the suspended sediment concentration distribution through the water column, which allowed for additional comparisons for these two formulas with some of the field cases for which concentration measurements were made. This will be discussed in a forthcoming paper (Larson et al., in preparation).

The six formulas are summarized in Table 1, where the formulas, coefficient values, and wave and beach conditions of the data originally used for verification of the formulas are listed. Further details regarding the equations are given in Appendix A. This is necessary because some variants of the formulas have appeared in the literature. In the following, a short background to the formulas is presented together with their main

Table 1
Longshore sediment transport formulas (for notation see Appendix A)

| Formula | Longshore sediment transport formula | Coefficients | Verification data | | |
|---|---|---|---|---|---|
| | | | $D$ (mm) | $\tan\beta$ | Exp. condition |
| Bijker | $q_{\mathrm{b,B}} = Ad_{50}\dfrac{V}{C}\sqrt{g}\,exp\left[\dfrac{-0.27(s-1)d_{50}\rho g}{\mu\tau_{\mathrm{b,wc}}}\right]$ | $A=1-5$ | 0.23 | 0.07 | $H_0=1.6$ m; |
| | $q_{\mathrm{s,B}} = 1.83q_{\mathrm{b,B}}\left[I_1\ln\left(\dfrac{33h}{r}\right)+I_2\right]$ | (non-breaking–breaking) | | | $T=12.0$ s; |
| | | | | | $\alpha=13°$ |
| Engelund–Hansen | $q_{\mathrm{t,EH}} = V\dfrac{0.05C\tau_{\mathrm{b,wc}}^2}{(s-1)^2 d_{50}\rho^2 g^{5/2}}$ | – | 0.19–0.93 | – | – |
| Watanabe | $q_{\mathrm{t,W}} = A\left[\dfrac{(\tau_{\mathrm{b,wc}}-\tau_{\mathrm{b,cr}})V}{\rho g}\right]$ | $A=0.5-2$ | 0.2–2.0 | 0.2–0.01 | $H_0=0.02-2.4$ m; |
| | | (regular–irregular) | | | $T=1.0-18.0$ s;<br>$\alpha=15-45°$ |
| Ackers–White | $q_{\mathrm{t,AW}} = V\dfrac{1}{1-p}d_{35}\left(\dfrac{V}{V_*}\right)^n\dfrac{C_{\mathrm{d,gr}}}{A^m}(F_C-A)^m$ | $A$, $m$, $n$, $C_{\mathrm{d,gr}}$, $F_C$ | 0.2–0.61 | – | $h=0.18-7.17$ m |
| (not modified) | | (see Appendix A) | | | |
| Van Rijn | $q_{\mathrm{b,VR}} = 0.25\gamma\rho_{\mathrm{s}}d_{50}D_*^{-0.3}\sqrt{\dfrac{\tau'_{\mathrm{b,wc}}}{\rho}}\left[\dfrac{\tau'_{\mathrm{b,wc}}-\tau_{\mathrm{b,cr}}}{\tau_{\mathrm{b,cr}}}\right]^{1.5}$ | – | 0.1–0.2 | – | $H_0=0.07-0.2$ m; |
| | | | | | $T=1.0-2.0$ s; |
| | $q_{\mathrm{s,VR}} = c_{\mathrm{a}}Vh\dfrac{1}{h}\displaystyle\int_a^h \dfrac{v}{V}\dfrac{c}{c_{\mathrm{a}}}dz = c_{\mathrm{a}}VhF$ | | | | $\alpha=90°$ |
| Bailard–Inman | $q_{\mathrm{t,BI}} = 0.5\rho f_{\mathrm{w}}u_0^3\dfrac{e_{\mathrm{b}}}{(\rho_{\mathrm{s}}-\rho)g\tan\gamma}\left(\dfrac{\delta_{\mathrm{v}}}{2}+\delta_{\mathrm{v}}^3\right)$ | $e_{\mathrm{b}}=0.1$; $e_{\mathrm{s}}=0.02$ | 0.175–0.6 | 0.034–0.138 | $H_0=0.05-1.44$ m; |
| | $+0.5\rho f_{\mathrm{w}}u_0^4\dfrac{e_{\mathrm{s}}}{(\rho_{\mathrm{s}}-\rho)gw_{\mathrm{s}}}(\delta_{\mathrm{v}}u_3^*)$ | | | | $T=1.0-11.0$ s; |
| | | | | | $\alpha=2.8-18.9°$ |

characteristics as well as references to the original publications. Also, Van De Graaff and Van Oveerem (1979) can be consulted for a comprehensive summary of some formulas. They compared three formulas for the net longshore sediment transport, namely the formulas by Bijker, Engelund–Hansen, and Ackers–White, although they focused on the gross rate and made comparisons for a number of selected hypothetical cases.

### 2.1. The Bijker formula

Bijker's (1967, 1971) sediment transport formula is one of the earliest formulas developed for waves and current in combination. It is based on a transport formula for rivers proposed by Kalinske–Frijlink (Frijlink, 1952). Bijker distinguishes between bed load and suspended load, where the bed load transport depends on the total bottom shear stress by waves and currents. The suspended load is obtained by integrating the product of the concentration and velocity profiles along the vertical, where the reference concentration for the suspended sediment is expressed as a function of the bed load transport. In its original form, the bed-load formula does not take into account a critical shear stress for incipient motion, implying that any bed shear stress and current will lead to a net sediment transport. The Bijker transport formula (hereafter, called the B formula) is, in principle, applicable for both breaking and non-breaking waves. However, different empirical coefficient values are needed in the formula.

### 2.2. The Engelund and Hansen formula

Engelund and Hansen (1967) originally derived a formula to calculate the bedload transport over dunes in a unidirectional current by considering an energy balance for the transport. Later, this formula (hereafter, called EH formula) was applied to calculate the total sediment transport under waves and currents, and modifications were introduced to account for wave stirring (Van De Graaff and Van Overeem, 1979). However, their theory has limitations when applied to graded sediments containing large amount of fine fractions, causing predicted transport rates to be smaller than the actual transport rates. Similar to the Bijker formula, no threshold conditions for the initiation of motion was included in the original formulation. The same coefficient values are used for monochromatic and random waves.

### 2.3. The Ackers and White formula

Ackers and White (1973) developed a total load sediment transport formula for coarse and fine sediment exposed to a unidirectional current. Coarse sediment is assumed to be transported as bed load with a rate taken to be proportional to the shear stress, whereas fine sediment is considered to travel in suspension supported by the turbulence. The turbulence intensity depends on the energy dissipation generated by bottom friction, which makes the suspended transport rate related to the bed shear stress. The empirical coefficients in the Ackers–White formula (hereafter, called AW formula) were calibrated against a large data set covering laboratory and field cases (HR Wallingford 1990; reported in Soulsby, 1997). Van De Graaff and Van Overeem (1979) modified the AW formula to account for shear exerted by waves.

### 2.4. The Bailard and Inman formula

Bailard and Inman (1981) derived a formula for both the suspended and bed load transport based on the energetics approach by Bagnold (1966). Bagnold assumed that the work done in transporting the sediment is a fixed portion of the total energy dissipated by the flow. The Bailard–Inman formula (hereafter, called BI formula) has frequently been used by engineers because it is computationally efficient, takes into account bed load and suspended load, and the flow associated with waves (including wave asymmetry) and currents can be incorporated in a straightforward manner. A reference level for the velocity employed in the formula (normally taken to be 5.0 cm above the bed) must be specified.

### 2.5. The Van Rijn formula

Van Rijn (1984) proposed a comprehensive theory for the sediment transport rate in rivers by considering both fundamental physics and empirical observations and results. The formulations were extended to estuaries as summarized by Van Rijn (1993) (hereafter,

called VR formula). Bed load and suspended load are calculated separately, and the approach of Bagnold (1966) is adopted for computing the bed load. Suspended load is determined by integrating the product of the vertical concentration and velocity profiles, where the concentration profile is calculated in three layers. Different exponential or power functions are employed in these layers with empirical expressions that depend on the mixing characteristics in each layer.

*2.6. The Watanabe formula*

Watanabe (1992) proposed a formula for the total load (bed load and suspended load) based on the power model concept. The volume of sediments set in motion per unit area is proportional to the combined shear stress of waves and currents, if the critical value for incipient motion is exceeded, and this volume is transported with the mean flow velocity. This formula has been widely used in Japan for the prediction of, for example, beach evolution around coastal structures and sand deposition in harbors and navigation channels. The Watanabe formula (hereafter, called W formula) and its coefficient values have been calibrated and verified for a variety of laboratory and field data sets during the last decade (e.g. Watanabe, 1987; Watanabe et al., 1991). However, it has not yet been established whether the value of the non-dimensional coefficient in the formula ($A$) is a constant or it depends on the wave and sediment conditions. Different values are employed for laboratory and field conditions, whereas the same value is typically used for monochromatic and random waves.

## 3. Longshore sediment transport data sets

*3.1. DUCK85 surf zone sand transport experiment*

The DUCK85 surf zone sand transport experiment was performed at the U.S. Army Corps of Engineers' Field Research Facility at Duck, NC in September, 1985. Kraus et al. (1989) measured the cross-shore distribution of the longshore sediment transport rate using streamer traps. Eight runs were made where the amount of sediment transported at a specific location in the surf zone during a certain time was collected using streamer traps oriented so that the traps opposed the direction of the longshore current. The traps, each consisting of a vertical array of polyester sieve cloth streamers suspended on a rack, were deployed across the surf zone. The polyester cloth allowed water to pass through but retained grains with diameter greater than the 0.105 mm mesh, which encompasses sand in the fine grain size region and greater. From knowledge of the trap mouth area, the trap efficiency, and the measurement duration, the local transport rate was derived. The trapping efficiency has been extensively investigated through laboratory experiments (Rosati and Kraus, 1988) allowing for confident estimates of the local longshore sediment transport rate.

Wave height and period were measured using the photopole method described by Ebersole and Hughes (1987). This method involved filming the water surface elevation at the poles placed at approximately 6-m intervals across the surf zone utilizing as many as eight 16-mm synchronized cameras. The bottom profile along the photopole line was surveyed each day. Fig. 1 shows surveyed bottom topography along the measurement transect on September 6, 1985. The root-mean-square (rms) wave height ($H_{rms}$) at the most offshore pole was in the range of 0.4–0.5 m, and the peak spectral period ($T_p$) was in the range of 9–12 s (see Table 2). Long-crested waves of cnoidal form arriving from the southern quadrant prevailed during the experiment, producing a longshore current moving to the north with a magnitude of 0.1–0.3 m/s.

*3.2. SUPERDUCK surf zone sand transport experiment*

Patterned after the DUCK85 experiment, the SUPERDUCK surf zone sand transport experiment was conducted during September and October 1986 (Rosati et al., 1990). However, at SUPERDUCK a temporal sampling method to determine transport rates was emphasized in which traps were interchanged from 3 to 15 times at the same locations. Fewer runs are available where the cross-shore distribution was measured (two runs were employed here; see Table 2).

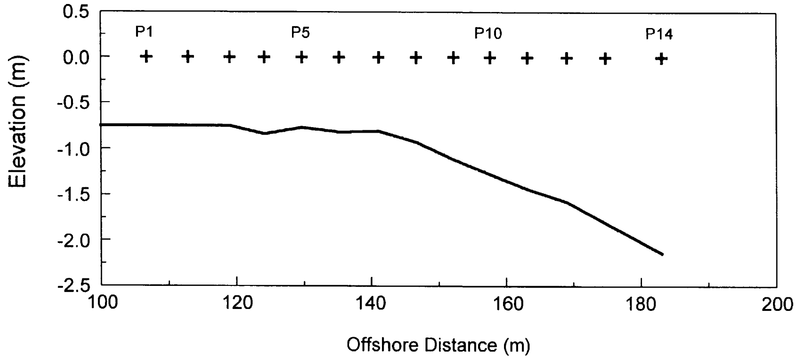


Fig. 1. Bottom profile surveyed along the measurement transect on September 6, 1985 (DUCK85 experiment).

Waves and currents were measured in the same manner as for DUCK85. The wave conditions during the experiment were characterized as long-crested swell ($T_p$ in the range 6–13 s) with a majority of the waves breaking as plunging breakers. The wave height ($H_{rms}$) at the most seaward pole varied between 0.3 and 1.6 m during the experiment. A steady offshore wind (6–7 m/s) was typically present during the measurements. The mean longshore current speeds measured were in the range of 0.1–0.7 m/s. During the experiment, the seabed elevation at each of the photopoles was surveyed once a day and Fig. 2 shows, as an example, the surveyed bottom profile on September 19, 1986. In contrast to the DUCK85 experiment, barred profiles occurred several times during the SUPERDUCK experiment, although the two runs presented here involved shelf-type profiles.

### *3.3. SANDYDUCK surf zone sand transport experiment*

The SANDYDUCK experiment was conducted in the same location as the DUCK85 and SUPERDUCK experiments (Miller, 1998). SANDYDUCK included several major storms, complementing the measurements made during DUCK85 and SUPERDUCK.

Table 2
Beach and wave characteristics for runs selected from the DUCK85, SUPERDUCK, and SANDYDUCK experiments for comparison with sediment transport formulas

| Date | Profile type | $H_{rms}$ (m) | $T_p$ (s) | $D_{ref}$ (m) | $V_{mean}$ (m/s) |
|---|---|---|---|---|---|
| Sept. 5, 1985, 09.57 | Shelf | 0.50 | 11.4 | 2.14 | 0.11 |
| Sept. 5, 1985, 10.57 | Shelf | 0.46 | 11.2 | 1.80 | 0.17 |
| Sept. 5, 1985, 13.52 | Shelf | 0.54 | 10.9 | 2.19 | 0.17 |
| Sept. 5, 1985, 15.28 | Shelf | 0.46 | 11.1 | 1.94 | 0.22 |
| Sept. 6, 1985, 09.16 | Shelf | 0.48 | 12.8 | 1.40 | 0.30 |
| Sept. 6, 1985, 10.18 | Shelf | 0.36 | 13.1 | 2.14 | 0.29 |
| Sept. 6, 1985, 13.03 | Shelf | 0.42 | 10.1 | 2.43 | 0.22 |
| Sept. 6, 1985, 14.00 | Shelf | 0.36 | 11.2 | 2.34 | 0.18 |
| Sept. 16, 1986, 11.16 | Shelf | 0.60 | 10.1 | 2.20 | 0.20 |
| Sept. 19, 1986, 10.16 | Shelf | 0.59 | 10.1 | 2.66 | 0.17 |
| March 31, 1997 | Bar | 1.36 | 8.0 | 6.70 | 0.49 |
| April 1, 1997 | Bar | 2.92 | 8.0 | 6.82 | 1.10 |
| October 20, 1997 | Bar | 2.27 | 12.8 | 6.44 | 0.53 |
| February 04, 1998 | Bar | 3.18 | 12.8 | 8.59 | 0.60 |
| February 05, 1998 | Bar | 2.94 | 12.8 | 6.83 | 0.45 |

$H_{rms}$ = root-mean-square wave height at the most offshore measurement point; $T_p$ = peak spectral period; $D_{ref}$ = water depth at the most offshore measurement point; $V_{mean}$ = mean longshore current velocity in the surf zone.

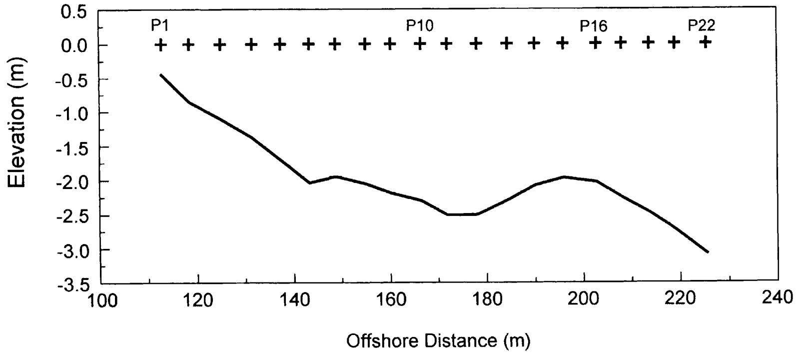


Fig. 2. Bottom profile surveyed along the measurement transect on September 19, 1986 (SUPERDUCK experiment).

Sediment transport measurements were made using the Sensor Insertion System (SIS), which is a diverless instrument deployment and retrieval system that can operate in seas with individual wave heights up to 5.6 m (Miller, 1999). An advantage of the SIS is that it allows direct measurement of wave, sediment concentration, and velocity together with the bottom profile during a storm. The SIS is a track-mounted crane with the instrumentation placed on the boom. A standard SIS consists of OBS to measure sediment concentration, an electromagnetic current meter for longshore and cross-shore velocities, and pressure gauge for waves and water levels. The wave conditions and mean longshore current velocities for five storm cases employed in this investigation are described in Table 2. During the SANDYDUCK experiment the concentration was measured at several points through the vertical as well as at a number of cross-shore locations. Simultaneously, the velocity was recorded and the local transport rate was derived from the product of the concentration and velocity.

Longshore bars were typically present during SANDYDUCK, making it possible to assess the effects of these formations on the transport rate distribution. As an example, the bottom profile measured during the storm on October 20, 1997, is shown in Fig. 3. The beach at Duck is composed of sand with a median grain size ($d_{50}$) of about 0.4 mm at the shoreline dropping off at a high rate to 0.18 mm in the offshore. In the region where $d_{50}=0.18$ mm (most of the typical surf zone width), sediment sampling has yielded $d_{35}=0.15$ mm (diameter corresponding 35%

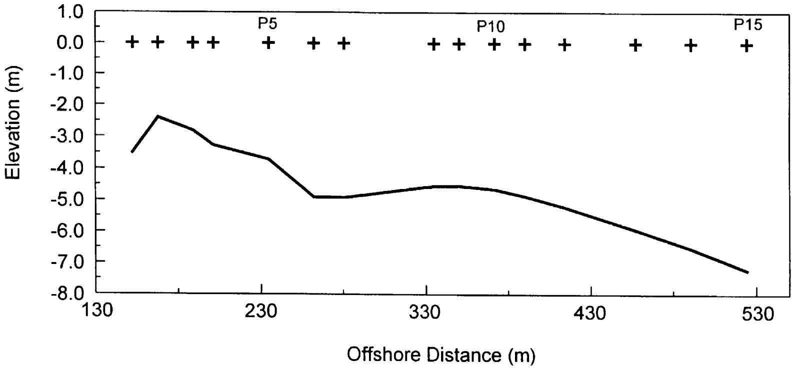


Fig. 3. Bottom profile surveyed along the measurement transect on October 20, 1997 (SANDYDUCK experiment).

being finer) and $d_{90} = 0.24$ mm (diameter corresponding 90% being finer) as typical values (Birkemeier et al., 1985).

## 4. Evaluation of the formulas

Measured hydrodynamics were employed as much as possible in the formulas to reduce the uncertainties in the transport calculations. The rms wave height and peak spectral wave period was used as the characteristic input parameters to quantify the random wave field. Values at intermediate locations where no measurements were made were obtained by linear interpolation (note that this is the cause for the discontinuities in the derivative of the calculated transport rate distributions). It was assumed that the incident wave angle was small, implying that the angle between the waves and the longshore current was approximately 90°. In the VR formula the undertow velocity is needed if the resultant shear stress is calculated (i.e., the shear stress resulting from the cross-shore and longshore currents combined). No undertow measurements were available and the model of Dally and Brown (1995) was employed to calculate this velocity. The influence of the shear stress from the undertow was typically small compared to that of the longshore current and waves. In a few cases extrapolation of the current from the most shoreward or seaward measurement point was needed. On the shoreward side the current was assumed to decrease linearly to become zero at the shoreline, whereas at the seaward end the current was taken to be proportional to the ratio of breaking waves (i.e., assuming that most of the current was wave-generated in this region).

The roughness height ($r$), which determines the friction factors for waves and current, is a decisive parameter that may markedly influence the sediment transport rate, especially the bed load transport. Here, the calculation of the roughness height was divided into three different cases depending on the bottom conditions, namely flat bed, rippled bed, and sheet flow. The division between these cases was made based on the Shields parameter ($\theta$), where $\theta < 0.05$ implied flat bed, $0.05 < \theta < 1.0$ rippled bed, and $\theta > 1.0$ sheet flow (Van Rijn, 1993). An iterative approach was needed because the bottom conditions are not known a priori when the roughness calculation is performed. A Shields curve was employed to determine the criterion for the initiation of motion based on $\theta$, which was included in the formulas that have this feature.

The roughness height was estimated in the following manner:

- *Flat bed*: $r$ is set equal to $2.5d_{50}$, where $d_{50}$ is the median grain size (Nielsen, 1992)
- *Rippled bed*: $r$ is calculated from the ripple height and length and the Shields parameter according to Nielsen (1992)
- *Sheet flow*: $r$ is calculated from the Shields parameter and $d_{90}$ according to Van Rijn (1984), where $d_{90}$ is the grain size that 10% of the sediment exceeds by weight.

The wave friction factor ($f_w$) was computed based on $r$ using the formula proposed by Swart (1976), which is based on an implicit relationship given by Jonsson (1966), assuming rough turbulent flow,

$$\begin{aligned} \ln(f_w) &= -5.98 + 5.2\left(\frac{r}{a_b}\right)^{0.194} && \text{for}: \frac{r}{a_b} < 0.63 \\ f_w &= 0.3 && \text{for}: \frac{r}{a_b} \geq 0.63 \end{aligned} \tag{1}$$

where $a_b$ is the amplitude of the horizontal near-bed water particle excursion. For the purpose of comparing the predictive capabilities of the formulas, coefficient values proposed by the developers and coworkers were employed without any particular tuning of the coefficients. Predicted transport rates with the formulas were converted to mass flux per unit width before comparison with the measurements.

### 4.1. Comparisons with DUCK85 data

Although simulations were carried out for all runs listed in Table 2, only four of the runs were selected for detailed discussion here. However, the overall conclusions given are based on the results from all simulations. Fig. 4 shows predicted cross-shore distributions of the longshore sediment transport rate with the different formulas for the experimental run at 0957 on September 5, 1985 (denoted 859050957

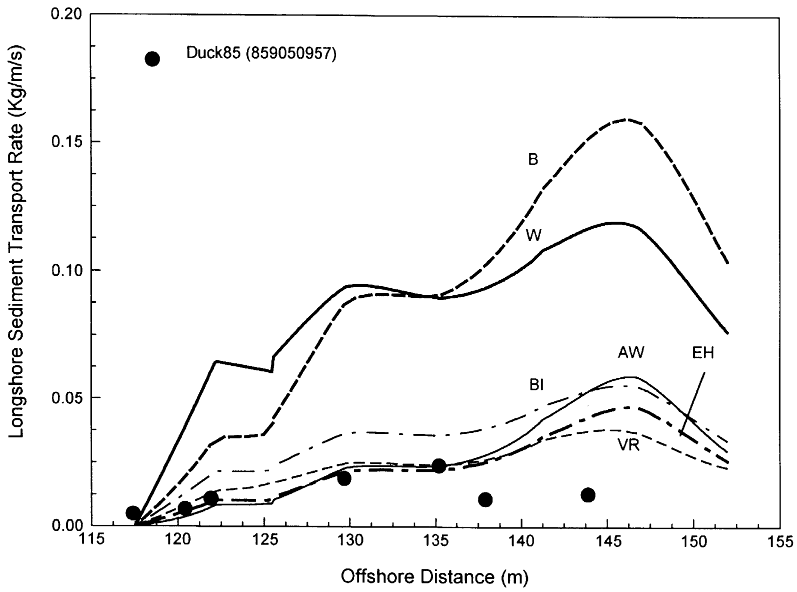


Fig. 4. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 859050957 from the DUCK85 experiment.

from year/month/day/time, in accordance with Kraus et al., 1989). It is noted that the AW, EH, and VR formulas yield good predictions within the same order of magnitude as the measured data, at least in the inner part of the surf zone. Contrarily, the B, BI, and W formulas give significantly higher transport rates than the measurements, especially B and W. Some overestimation is expected for the B and BI formulas because they do not take into account the threshold of sediment motion in their original formulations, although this simplification would not account for the large deviations found for the B formula. Bijker (1971) pointed out that his formula tended to overestimate the transport rate using the recommended value on the main coefficient.

Figs. 5 and 6 show calculated and measured transport rates for the experimental runs 859051528 and 859060916, respectively. All formulas except B and W predicted longshore transport rates at the correct order of magnitude, although the EH formula somewhat underpredicted the transport rate. Bearing in mind that the sediment has a $d_{50}$ of approximately 0.18 mm at the site, the discrepancy between the EH formula and the measured rates might be due to limitations in the derivation of the formula. For fine-graded sediments large suspension modifies the velocity distribution so that the assumptions underlying the formula are not satisfied (Engelund and Hansen, 1967). Another reason for the discrepancy might be the relatively strong influence of the predicted roughness on the transport rate that the EH formula displays. Run 859051528 (see Fig. 5) indicated a bimodal distribution with a large peak in the outer surf zone (at around 145 m) and a small peak in the inner surf zone (at around 125 m). In general, the formulas fail to correctly predict this shape, especially the shoreward peak. This peak is probably a function of additional breaking and current generation close to the shoreline. Because no current measurements were available here, extrapolation was employed, implying that some portion of the discrepancy is probably caused by the uncertainty in the input data rather than the formulas themselves.

During Run 859060916 the most seaward trap was located outside the surf zone and a relatively small amount of sand was collected in it (Fig. 6). Thus, there is clear evidence that the longshore transport rate dropped off steeply seaward of the break point. Predictions by the AW, BI, EH, and VR formulas were in satisfactory agreement with the measured

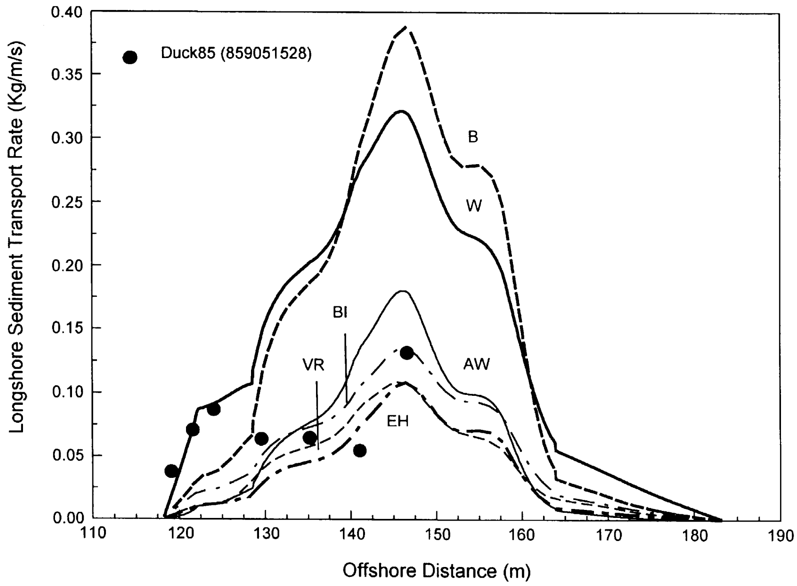


Fig. 5. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 859051528 from the DUCK85 experiment.

rates both inside and outside the surf zone. In Fig. 7, predictions of the transport rates are compared to measurements from Run 859061018, and all formulas overpredict the rates outside the break point (approximately located at 140 m). In contrast, the AW, BI, EH, and VR formulas produced satisfactory results in the surf zone, whereas B and W produce much too large transport rates in this zone as well.

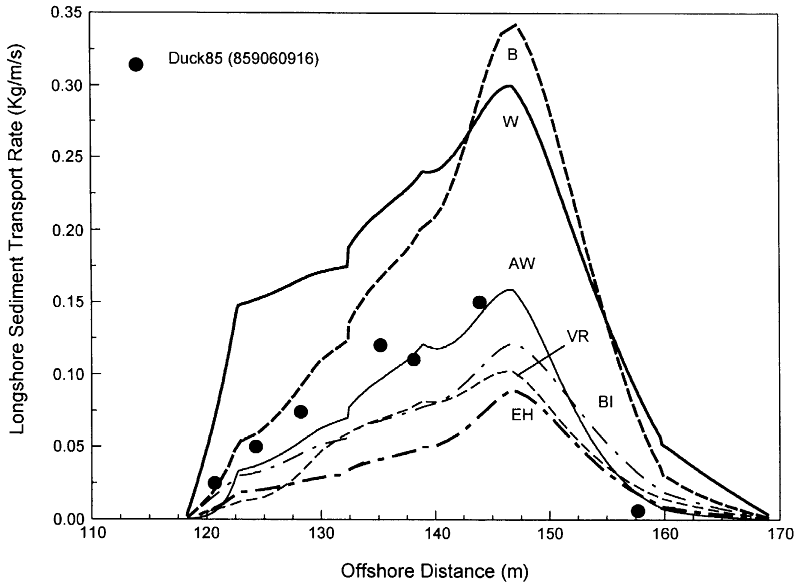


Fig. 6. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 859060916 from the DUCK85 experiment.

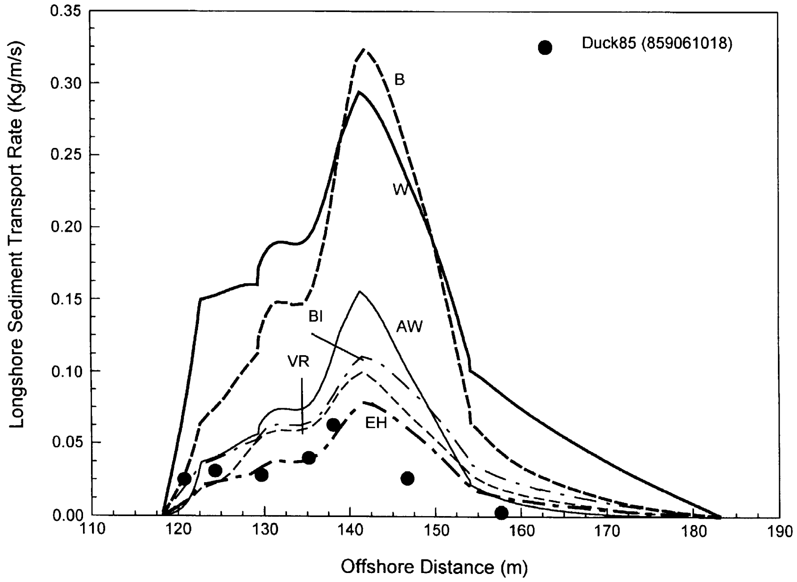


Fig. 7. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 859061018 from the DUCK85 experiment.

In general, based on the calculation results from the DUCK85 runs summarized in Table 2, the B, BI, and W formulas overestimated the transport, whereas the AW, EH, and VR formulas yielded overall good predictions. Most of the formulas produced cross-shore distributions that were more or less in agreement with the measured distributions, although some calibration factors might be needed to achieve quantitative agreement. The observed discrepancy between the measurements and predictions using standard coefficient values is attributed to several factors: all formulas rely on a considerable number of parameters and coefficients, where the values were typically determined from situations not completely representative for the field (e.g., laboratory, river environment). Also, the transport is sensitive to the estimated bottom roughness, which is a difficult quantity to determine, thereby introducing significant uncertainty into the calculations.

## 4.2. Comparisons with SUPERDUCK data

Four runs were conducted during the SUPERDUCK experiment where the cross-shore distribution of the longshore sand transport rate was measured. However, only two runs had sufficient information on the local hydrodynamics to allow comparison with the predictive formulas. Figs. 8 and 9 show the measured and computed longshore transport rates for Runs 8609161116 and 8609191016, respectively. For Run 8609161116, similar conclusions can be drawn as for the earlier discussed DUCK85 runs concerning all the formulas. The B and W formulas overestimate the transport rates, whereas the AW, BI, EH, and VR formulas produce distributions that are in good agreement with the measurements, at least shoreward of the main break point. A relatively small amount of sand was collected outside the surf zone (break point located at approximately 170 m; see Fig. 8), again displaying the sharp drop in the sand transport rate occurring seaward of the break point. For Run 8609191016 (Fig. 9) the formulas showed an agreement with the data in accordance with the previous runs, although the over-predictions were relatively more marked outside the surf zone for this run.

## 4.3. Comparisons with SANDYDUCK data

Five experimental runs from the SANDYDUCK experiment (Miller, 1998) were available for compar-

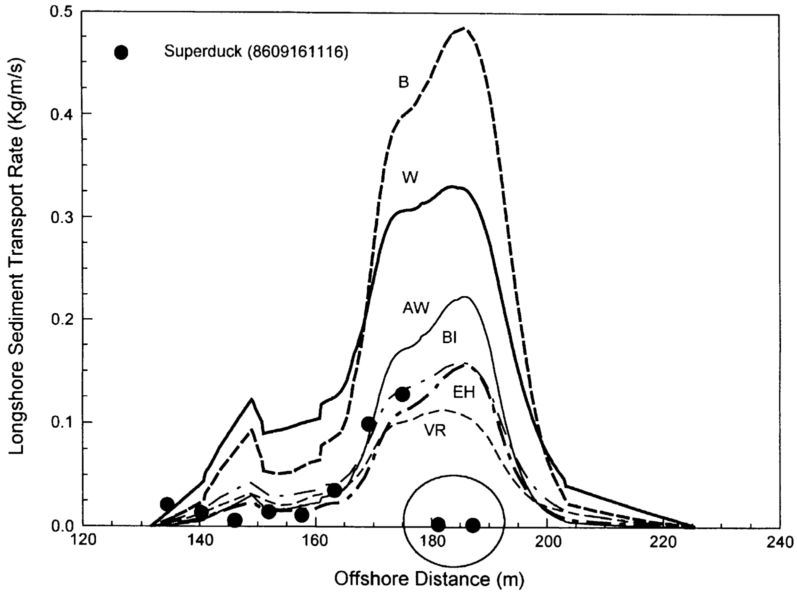


Fig. 8. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 8609161116 from the SUPERDUCK experiment.

ison with the predictive formulas. As opposed to the DUCK85 and SUPERDUCK experiments, the transport rates were found to be in the sheet flow regime for all SANDYDUCK measurements. The cross-shore distribution of the measured and calculated transport rates on March 31, 1997 (denoted 97/03/31) is given

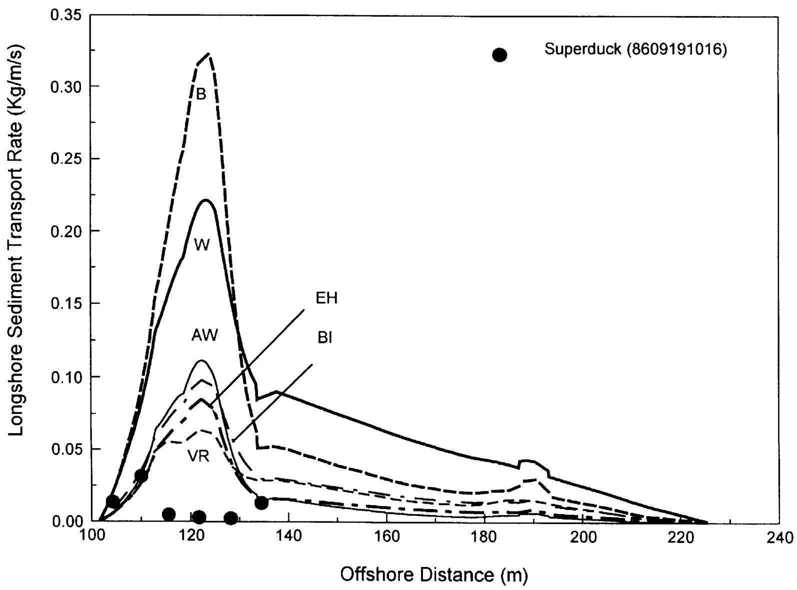


Fig. 9. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 8609191016 from the SUPERDUCK experiment.

in Fig. 10. For this run, the predicted peak transport rates are markedly shifted shoreward relative to the measured peak. Thus, all formulas produce unsatisfactory results, although the BI, EH, VR, and W formulas yield values that are more in agreement with the data than the AW and B formulas. In Fig. 11 comparisons are made between predictions and measurements for the run on April 1, 1997. As for Run 97/03/31, the BI, EH and W formulas capture the main features of the measured transport rate distribution. Consequently, these formulas give better predictions under sheet flow conditions than if low-energy swell waves prevail, which was the case for DUCK85 and SUPERDUCK. The AW and B formulas have a tendency to overpredict under conditions giving large measured transport rates, especially the AW formula. This is partly because under high waves in the surf zone the transport is dominated by suspended load. As shown by Larson et al. (in preparation), the formulas typically overestimate the time-averaged sediment concentration, implying that the total transport rate becomes too large.

Fig. 12 shows predicted and measured longshore transport rates for a storm on October 20, 1997. The measured transport rate distribution across shore is bimodal with one peak just shoreward of the break point and the other peak near the shoreline. However, there is a large variability in the measured rate, both at the scale of the surf zone and in between points, which is difficult to explain in terms of the measured forcing. Calculations with the AW, BI, EH, VR, and W formulas yield acceptable agreement with the measured distribution in most of the surf zone, but near the shoreline the measured peak is not predicted. The measured wave heights and currents at the points closest to the shoreline do not indicate a potential for large transport here. The AW and B formulas overpredict the transport rate in the outer part of the surf zone, but are doing better in the inner part, as well as outside the break point.

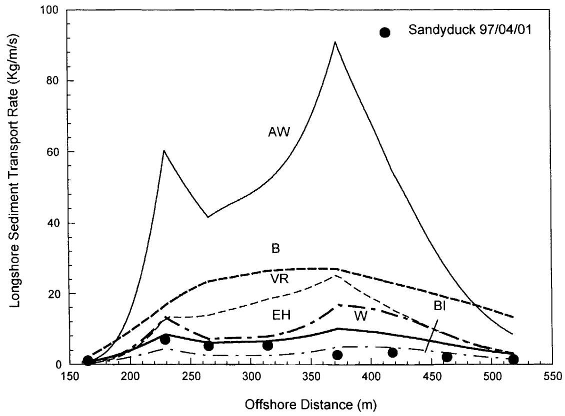


Fig. 11. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 97/04/01 from the SANDYDUCK experiment.

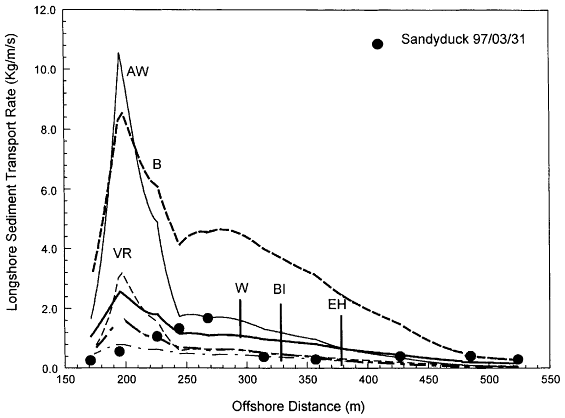


Fig. 10. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 97/03/31 from the SANDYDUCK experiment.

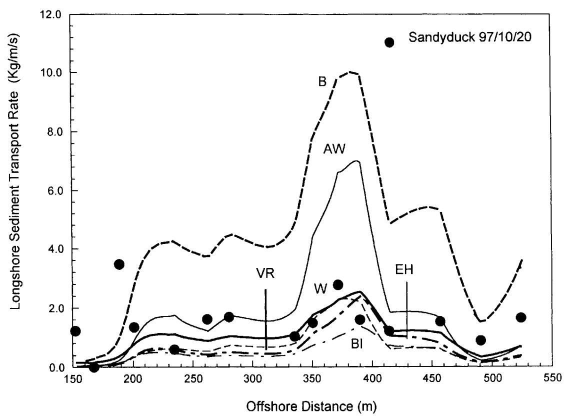


Fig. 12. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 97/10/20 from the SANDYDUCK experiment.

Fig. 13 shows calculated and measured transport rate distributions for the run on February 4, 1998, which is representative for large transport on a barred profile during a storm. The peak in the transport rate was observed some distance shoreward of the bar crest, whereas the formulas predicted the peak to occur more seaward (i.e., close to the bar crest). In this respect, the BI, EH, and W formulas yield locations of the peak transport that are more seaward than predicted by AW, B, and VR. In contrast to Run 98/02/04, Fig. 14 shows a comparison for Run 98/02/05 representative of the transport during a moderate storm. Most of the formulas underpredict the transport rate, however, the AW and B formulas are yielding satisfactory agreement. The general features of the cross-shore distribution are well reproduced by all formulas, indicating that tuning of the coefficients in the formulas would considerably improve the predictions.

Comparisons between the SANDYDUCK measurements and the formulas allowed for an evaluation of their predictive capability during storm conditions. Overall, the AW and B formulas predicted higher transport rates than the other formulas as well as the measurements. Also, the VR and W formulas yielded slightly better predictions than the BI and EH formulas. The formulas often failed to accurately predict the location of the peak in the transport rate in the surf zone. However, seaward of this peak, all formulas except B displayed satisfactory agreement with the measured rates.

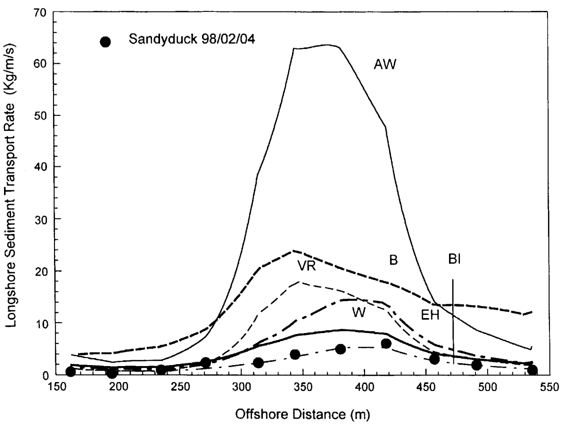

Fig. 13. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 98/02/04 from the SANDYDUCK experiment.

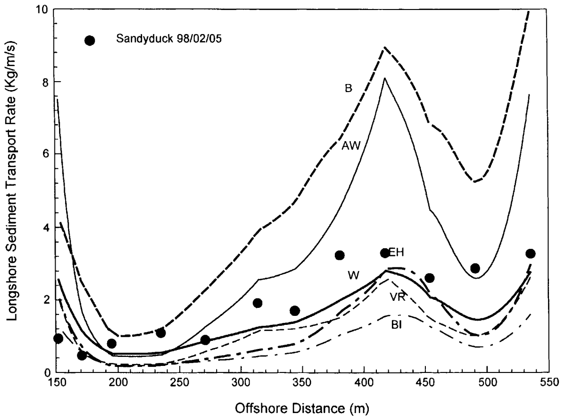

Fig. 14. Comparison between calculated and measured cross-shore distribution of longshore sediment transport rate for Run 98/02/05 from the SANDYDUCK experiment.

## 5. Discussion of results

Comparisons between field measurements and calculations indicated that several of the formulas yield predictions that might be considered acceptable in many coastal engineering applications. However, to objectively quantify the predictive capability of the formulas, an overall comparison based on individual point measurements was carried out for each formula. Fig. 15 summarizes this comparison for the six selected formulas and all measurements from the three data sets. In the figure, the circles denote data points from the DUCK85 and SUPERDUCK experiments, representative of transport under low-energy conditions, and the squares denote points from the SANDYDUCK experiment, indicative of the transport during storm conditions. Viewing Fig. 15, the BI and VR formulas show best agreement regarding the DUCK85 and SUPERDUCK data, displaying least scatter around the line of perfect agreement (i.e., the ratio between predicted and measured transports, $q_p/q_m$, respectively, is one). Most of the computed values are within a factor 5 of the measured values (see dashed lines in Fig. 15) for these two formulas. The B and W formulas have a tendency to overpredict the

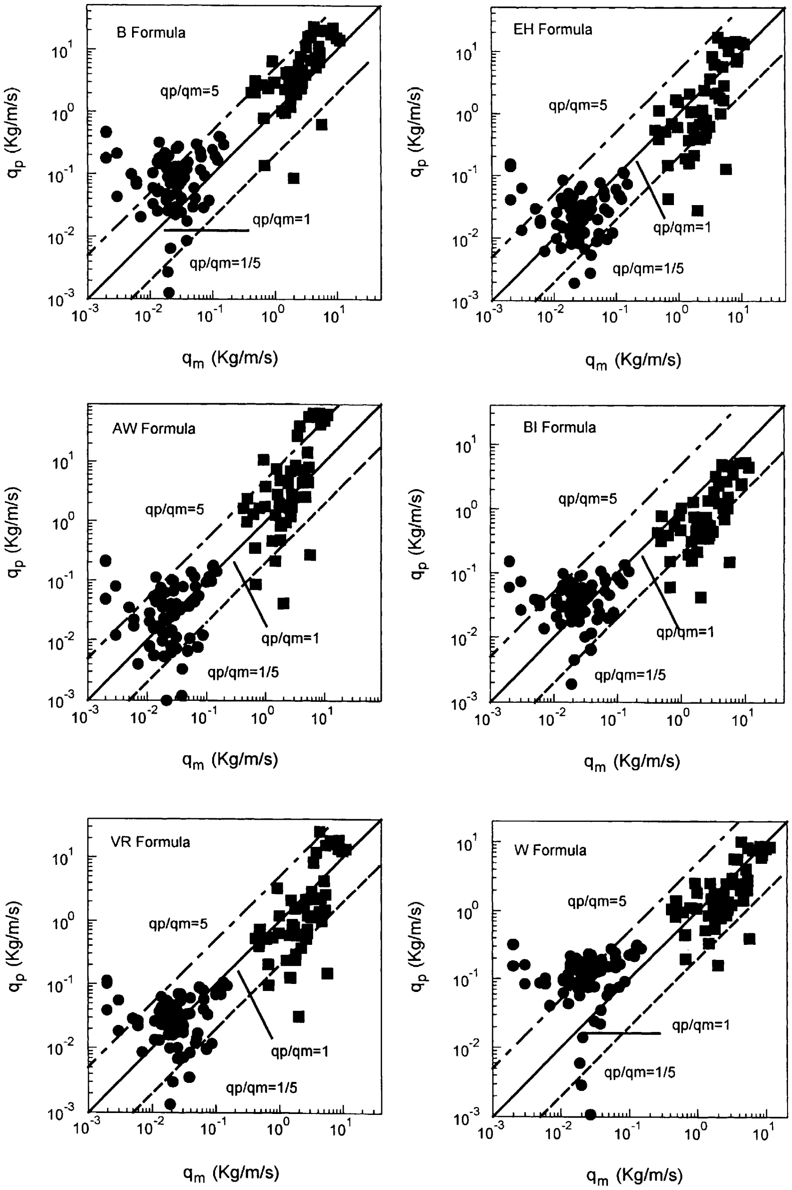


Fig. 15. Comparison between calculated and measured cross-shore distribution of longshore sediment transport for all three data sets employed.

measured transport rates, whereas the AW and EH formulas show considerably more scatter around the line $q_p/q_m = 1.0$. Concerning the SANDYDUCK data, computed values with the AW and B formulas are typically too large and with the BI and EH formulas too small. The VR and W formulas yield the least scatter around the line of perfect agreement.

Quantitative and qualitative comparisons were made between measurements and predictions regarding the scatter, trend, and clustering of the calculated

Table 3
Summary of accuracy of all the formulas

| Formula | Scatter | | Trend | Clustering | Data with discrepancy ratio distribution between 1/5 and 5 | |
|---|---|---|---|---|---|---|
| | (DUCK85 + SUPERDUCK) | (SANDYDUCK) | | | (DUCK85 + SUPERDUCK) | (SANDYDUCK) |
| | $\sigma_{rms}$ | | | | (%) | |
| Bijker | 0.868 | 0.608 | 2 | 1 | 32 | 8 |
| Engelund–Hansen | 0.705 | 0.519 | 4 | 3 | 29 | 18 |
| Ackers–White | 0.812 | 0.724 | 4 | 3 | 20 | 22 |
| Bailard–Inman | 0.659 | 0.485 | 2 | 4 | 16 | 24 |
| Van Rijn | 0.662 | 0.518 | 3 | 4 | 19 | 16 |
| Watanabe | 0.864 | 0.349 | 2 | 1 | 38 | 4 |

points around $q_p/q_m = 1.0$. As a measure of the scatter, the rms error was calculated according to,

$$\sigma_{rms} = \left[ \frac{\sum_{1}^{N} \left[\log(q_p) - \log(q_m)\right]^2}{N-1} \right]^{1/2} \tag{2}$$

where $N$ is the number of data points. The computed $\sigma_{rms}$ values for all formulas are listed in Table 3, where a smaller $\sigma_{rms}$ value implies a smaller scatter. From the table it can be seen that the BI formula shows the smallest scatter for the DUCK85 and SUPERDUCK data, followed by the VR and EH formulas. The W formula shows the smallest scatter for the SANDYDUCK data, followed by the BI and VR formula. Taking an average for all data, the VR and BI formulas display the least scatter.

Based on visual observations (Fig. 15), the formulas were subjectively ranked from 1 (i.e., weak) to 5 (i.e., strong) concerning trends and clustering (see Table 3). Also, a relative rating of the predictions was assigned to the formulas utilizing a mean discrepancy ratio, given by the percentage of the measurement points lying between 1/5 to 5 of the predictions by the formulas (this value was subtracted from 100% to yield a small number for good agreement). The BI formula produce the smallest discrepancy ratio (16%) for DUCK85 and SUPERDUCK experiments, followed by the VR and AW formulas (19% and 20%, respectively). For the SANDYDUCK cases, the W formula has a discrepancy ratio of only 4% with the BI and VR formulas yielding ratios of 8% and 16%, respectively. Taking an average for all experimental cases, the VR formula produces the lowest discrepancy ratio, whereas the other formulas yield comparable ratios.

## 6. Conclusions

The VR formula gave the most reliable predictions over the entire range of wave conditions (swell and storm) studied, based on criteria involving the scatter, trend, and clustering of the predictions around the measurements. The AW formula gave satisfactory results for all conditions, but scatter was marked both for swell and storm. Regarding the scatter, the BI formula yielded improved predictions compared to AW, although the transport was systematically overestimated during swell and underestimated during storm. The EH formula displayed similar tendency as the AW formula, producing reasonable results over the entire range of wave conditions investigated, but displaying significant scatter. The W formula yielded the best predictions for the storm conditions, but markedly overestimated the transport rates for swell waves. Finally, the B formula systematically overestimated the transport rates for all conditions.

The coefficient values in the sediment transport formulas employed were the original ones as recommended by the authors. In most cases, these values were derived based upon laboratory data or data from a river environment, involving no or limited field

measurements pertaining to longshore sediment transport. Thus, additional calibration of the formulas against the available data sets would increase their predictive capability, but the modifications would be weighted by the particular data sets. For example the field data considered here only encompassed one median grain size (0.18 mm; i.e., fine sand).

At the present time, there is no well-established transport formula that takes into account all the different factors that control longshore sediment transport in the surf zone, although the VR evidently accounts for many of those factors. A complete formula should quantify bed load and suspended load, describe random waves as well as the effects of wave breaking, and include transport in the swash zone.

**Acknowledgements**

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## Appendix A. Longshore sediment transport formulas

### *A.1. Bijker formula (1967, 1971)*

Bijker (1967) modified the Kalinske–Frijlink formula (Frijlink, 1952) for bed load together with Einstein's method for evaluating the suspended load transport to be applied in a coastal environment. Thus, Bijker's formula, popular among European engineers, takes into account both waves and currents. The bed load transport rate ($q_{b,B}$; in m$^3$/s/m, including pores) is calculated from,

$$q_{b,B} = Ad_{50}\frac{V}{C}\sqrt{g}\,\exp\left[\frac{-0.27(s-1)d_{50}\rho g}{\mu\tau_{b,wc}}\right]$$

where $A$ is an empirical coefficient (1.0 for non-breaking waves and 5.0 for breaking waves), $d_{50}$ the median particle diameter, $V$ the mean longshore current velocity, $C$ the Chezy coefficient based on $d_{50}$, $g$ the acceleration of gravity, $s$ $(=\rho_s/\rho)$ the relative sediment density, $\rho_s$ the density of the bed material, $\rho$ the density of water, $\mu$ a ripple factor, and $\tau_{b,wc}$ the bottom shear stress due to the waves and current. The first part of the above expression represents a transport parameter, whereas the second part (the exponent) is a stirring parameter. The ripple factor, which indicates the influence of the form of the bottom roughness on the bed load transport, is expressed as,

$$\mu = \left(\frac{C}{C_{90}}\right)^{1.5}$$

where $C_{90}$ is the Chezy coefficient based on $d_{90}$, which is the particle diameter, exceeded 10% by weight. The combined shear stress at the bed ($\tau_{b,wc}$) induced by waves and current is (valid for a 90° angle between the waves and current),

$$\tau_{b,wc} = \tau_{b,c}\left[1 + \frac{1}{2}\left(\xi\frac{u_0}{V}\right)^2\right]$$

in which $\tau_{b,c}$ is the bed shear stress due to current only and $u_o$ the maximum wave orbital velocity near the bed. The coefficient $\xi$ is given by,

$$\xi = C\sqrt{\frac{f_w}{2g}}$$

in which $f_w$ is the wave friction factor (Jonsson, 1966).

To calculate the suspended load, Bijker (1967) assumed that the bedload transport occurred in a bottom layer having a thickness equal to the bottom roughness ($r$). The concentration of material in the bed load layer ($c_b$; assumed to be constant over the thickness) is:

$$c_b = \frac{q_{b,B}}{6.34\sqrt{\frac{\tau_{b,c}}{\rho}}\,r}$$

The concentration distribution is obtained from,

$$c(z) = c_b\left[\frac{r}{h-r}\,\frac{h-z}{z}\right]^{\frac{w\sqrt{\rho}}{\kappa\sqrt{\tau_{b,wc}}}}$$

where $z$ is the elevation, $h$ the water depth, $w$ the sediment fall speed, and $\kappa$ von Karman's constant. By

integrating along the vertical from the reference height to the water surface, the total suspended sediment load is determined as,

$$q_{\mathrm{s,B}} = 1.83 q_{\mathrm{b,B}} \left[ I_1 \ln\left(\frac{33h}{r}\right) + I_2 \right]$$

where $I_1$ and $I_2$ are the Einstein integrals (e.g., Van Rijn, 1993). The total load is computed as the sum of bed load and suspended load ($q_{\mathrm{t,B}} = q_{\mathrm{b,B}} + q_{\mathrm{s,B}}$).

*A.2. Engelund and Hansen (1967) formula*

Engelund and Hansen (1967) developed a formula to compute the bed load transport under a current. This formula was later used to compute the total load and also modified to take into account wave stirring. Applied to calculate the longshore sediment transport, the formula yields:

$$q_{\mathrm{t,EH}} = V \frac{0.05 C \tau_{\mathrm{b,c}}^2 \left[ 1 + \frac{1}{2}\left(\xi \frac{u_0}{V}\right)^2 \right]^2}{(s-1)^2 d_{50} \rho^2 g^{5/2}}$$

This formula is also composed of a stirring term and a transporting term, much in accordance with Watanabe (1992). The same coefficient value (= 0.05) apply for both monochromatic and random waves in the original formula.

*A.3. Ackers and White (1973) formula*

Similarly to Engelund and Hansen (1967), the formula proposed by Ackers and White (1973) initially predicted the total load transport under a current, but was later enhanced by Van De Graaff and Van Overeem (1979) to describe the effects of waves. The original Ackers–White formula may be written,

$$q_{\mathrm{t,AW}} = V \frac{1}{1-p} d_{35} \left(\frac{V}{V_*}\right)^n \frac{C_{\mathrm{d,gr}}}{A^m} (F_C - A)^m$$

where $p$ is the porosity of the sediment, $d_{35}$ the particle diameter exceeded by 65% of the weight, $V^*$ the shear velocity due to the current, $n$, $m$, $C_{\mathrm{d,gr}}$, and $A$ dimensionless parameters, and $F_C$ a sediment mobility number. The dimensionless parameters are written, respectively,

$$n = 1 - 0.2432 \ln(d_{\mathrm{gr}})$$

$$m = \frac{9.66}{d_{\mathrm{gr}}} + 1.34$$

$$C_{\mathrm{d,gr}} = \exp\left(2.86 \ln(d_{\mathrm{gr}}) - 0.4343[\ln(d_{\mathrm{gr}})]^2 - 8.128\right)$$

$$A = \frac{0.23}{\sqrt{d_{\mathrm{gr}}}} + 0.14$$

where,

$$d_{\mathrm{gr}} = d_{35} \left(\frac{g(s-1)}{v^2}\right)^{1/3}$$

and $v$ is the kinematic viscosity. The sediment mobility number is defined as,

$$F_C = \frac{V \left(\frac{V_*}{V}\right)^n C_{\mathrm{d}}^n}{C_{\mathrm{d}} g^{n/2} \sqrt{(s-1) d_{35}}}$$

in which:

$$C_{\mathrm{d}} = 18 \log\left(\frac{10h}{d_{35}}\right)$$

The modified equation by Van De Graaff and Van Overeem (1979) to take into account waves is written,

$$q_{\mathrm{t,AWM}} = V \frac{1}{1-p} d_{35} \left\{\frac{V'_{\mathrm{wc}}}{V_{*,\mathrm{wc}}}\right\}^n \frac{C_{\mathrm{d,gr}}}{A^m} \times \left\{ \frac{V'_{\mathrm{wc}} \left\{\frac{V_{*,\mathrm{wc}}}{V'_{\mathrm{wc}}}\right\}^n C_{\mathrm{d}}^n}{C_{\mathrm{d}}\, g^{n/2} \sqrt{(s-1) d_{35}}} - A \right\}^m$$

where,

$$V_{*,\mathrm{wc}} = V_* \left[ 1 + \frac{1}{2}\left(\xi \frac{u_0}{V}\right)^2 \right]^{1/2}$$

and:

$$V'_{\mathrm{wc}} = V \left[ 1 + \frac{1}{2}\left(\xi' \frac{u_0}{V}\right)^2 \right]^{1/2}$$

In the above formulation, $\xi'$ is based on $d_{35}$ and $\xi$ on the bed roughness $r$.

### A.4. Bailard and Inman (1981, 1984) formula

Bailard and Inman (1981) extended the formula introduced by Bagnold to oscillatory flow in combination with a steady current over a plane sloping bottom. The instantaneous bed load ($q'_{\mathrm{b,BI}}$) and suspended load ($q'_{\mathrm{s,BI}}$) transport rate vectors are expressed as,

$$q'_{\mathrm{b,BI}} = \frac{0.5 f_{\mathrm{w}} \rho e_{\mathrm{b}}}{(\rho_{\mathrm{s}} - \rho) g \tan\gamma} \left[ \left| U'_{\mathrm{t}} \right|^2 U'_{\mathrm{t}} - \frac{\tan\beta}{\tan\gamma} \left| U'_{\mathrm{t}} \right|^3 i\beta \right]$$

$$q'_{\mathrm{s,BI}} = \frac{0.5 f_{\mathrm{w}} \rho e_{\mathrm{s}}}{(\rho_{\mathrm{s}} - \rho) g w} \left[ \left| U'_{\mathrm{t}} \right|^3 U'_{\mathrm{t}} - \frac{e_{\mathrm{s}}}{w} \tan\beta \left| U'_{\mathrm{t}} \right|^5 i\beta \right]$$

in which $\tan\beta$ is the local bottom slope, $\tan\gamma$ a dynamic friction factor, $U_{\mathrm{t}}'$ the instantaneous velocity vector near the bed (wave and current) and $i_\beta$ is a unit vector in the direction of the bed slope. Averaging over a wave period, the total transport rate and direction are obtained containing both the wave- and current-related contributions. Assuming that a weak longshore current prevails, neglecting effects of the slope term on the total transport rate for near-normal incident waves, the local time-averaged longshore sediment transport rate is (Bailard, 1984),

$$q_{\mathrm{t,BI}} = 0.5 \rho f_{\mathrm{w}} u_0^3 \frac{e_{\mathrm{b}}}{(\rho_{\mathrm{s}} - \rho)\, g \tan\gamma} \left( \frac{\delta_{\mathrm{v}}}{2} + \delta_{\mathrm{v}}^3 \right) + 0.5 \rho f_{\mathrm{w}} u_0^4 \frac{e_{\mathrm{s}}}{(\rho_{\mathrm{s}} - \rho)\, g w_{\mathrm{s}}} (\delta_{\mathrm{v}} u_3^*)$$

where $e_{\mathrm{b}}$ and $e_{\mathrm{s}}$ are efficiency factors, and:

$$\delta_{\mathrm{v}} = \frac{V}{u_0}$$

$$u_3^* = \frac{\langle |U'_{\mathrm{t}}|^3 \rangle}{u_0}$$

The following coefficient values are typically used in calculations: $e_{\mathrm{b}} = 0.1$, $e_{\mathrm{s}} = 0.02$, $\tan\gamma = 0.63$. Thus, the efficiency factors are assumed to be constant, although work has indicated that $e_{\mathrm{b}}$ and $e_{\mathrm{s}}$ are related to the bed shear stress and the particle diameter. It should also be noted that the formula is derived for plane bed conditions.

### A.5. Van Rijn (1984, 1993) formula

Van Rjin (1984) presented comprehensive formulas for calculating the bed load and suspended load, and only a short description of the method is given in the following. For the bed load he adapted the approach of Bagnold assuming that sediment particles jumping under the influence of hydrodynamic fluid forces and gravity forces dominate the motion of the bed load particles. The saltation (jumps) characteristics were determined by solving the equation of motion for an individual sediment particle. The bed load can be defined as the product between the particle concentration ($c_{\mathrm{b}}$; a reference concentration for the bed load different from the reference concentration for suspended load $c_{\mathrm{a}}$), the particle velocity ($u_{\mathrm{b}}$), and the layer thickness ($\delta_b$; taken to be equal to the reference level $a$) according to,

$$q_{\mathrm{b,VR}} = c_{\mathrm{b}} u_{\mathrm{b}} \delta_{\mathrm{b}}$$

where,

$$\frac{c_{\mathrm{b}}}{c_0} = 0.18 \frac{T}{D_*}$$

$$D_* = d_{50} \left[ \frac{(s-1) g}{\nu^2} \right]^{1/3}$$

$$T = \frac{\tau'_{\mathrm{b,wc}} - \tau_{\mathrm{b,cr}}}{\tau_{\mathrm{b,cr}}}$$

in which $c_0$ ($=0.65$) is the maximum bed load concentration, $D_*$ the dimensionless grain diameter, $T$ the excess bed shear stress parameter, and $\tau'_{\mathrm{b,wc}}$ is the effective bed shear stress for waves and current combined (calculated according to Van Rijn's own method, not discussed here). Substituting the above expressions into the bed load transport formula together with some other relationships not given yields,

$$q_{\mathrm{b,VR}} = 0.25 \gamma \rho_{\mathrm{s}} d_{50} D_*^{-0.3} \sqrt{\frac{\tau'_{\mathrm{b,wc}}}{\rho}} \left[ \frac{\tau'_{\mathrm{b,wc}} - \tau_{\mathrm{b,cr}}}{\tau_{\mathrm{b,cr}}} \right]^{1.5}$$

where,

$$\gamma = 1 - \sqrt{\frac{H_{\mathrm{s}}}{h}}$$

in which $H_s$ is the significant wave height. The depth-integrated suspended load transport in the presence of current and waves is defined as the integration of the product of velocity ($v$) and concentration ($c$) from the edge of the bed-load layer ($z=a$) to the water surface, yielding:

$$q_{s,VR} = \int_a^h vcdz$$

Integrating after substituting in the longshore current can be shown to give,

$$q_{s,VR} = c_a Vh \frac{1}{h} \int_a^h \frac{v}{V} \frac{c}{c_a} dz = c_a VhF$$

where $c$ is the concentration distribution, $V$ the mean longshore current, and,

$$F = \frac{V_*}{\kappa V}\left(\frac{a}{h-a}\right)^Z \left( \int_{a/h}^{0.5} \left(\frac{h-z}{z}\right)^{Z'} \ln(z/z_0)d(z/h) + \int_{0.5}^{1} e^{-4Z'(z/h-0.5)} \ln(z/z_0)d(z/h) \right)$$

$$c_a = 0.015 \frac{d_{50}}{a} \frac{T^{1.5}}{D_*^{0.3}}$$

$$Z' = Z + \psi$$

$$Z = \frac{w}{\beta \kappa V_*}$$

$$\Psi = 2.5\left(\frac{w}{V_*}\right)^{0.8}\left(\frac{c_a}{c_0}\right)^{0.4}$$

$$\beta = 1 + 2\left(\frac{w}{V}\right)^2$$

in which $Z$ is a suspension parameter reflecting the ratio of the downward gravity forces and upward fluid forces acting on a suspended sediment particle in a current, $\psi$ is an overall correction factor representing damping and reduction in particle fall speed due to turbulence, and $\beta$ is a coefficient quantifying the influence of the centrifugal forces on suspended particles.

Van Rijn (1984) calculated the concentration distribution $c$ in three separate layers, namely:

- from the reference level $a$ to the end of a near-bed mixing layer (of thickness $\delta_s$)
- from the top of the $\delta_s$-layer to half the water depth ($h/2$)
- from ($h/2$) to $h$

Different exponential or power functions are employed in these regions with empirical expressions depending on the mixing characteristics in each layer.

### A.6. Watanabe (1992) formula

The formula proposed by Watanabe (1992) for the total load was developed to calculate the longshore sediment transport rate as combined bed and suspended load according to,

$$q_{t,W} = A\left[\frac{(\tau_{b,wc} - \tau_{b,cr})V}{\rho g}\right]$$

where $A$ is an empirical coefficient (about 0.5 for monochromatic waves and 2.0 for random waves) and $\tau_{b,cr}$ is the critical bed shear stress for incipient motion (determined from the Shield curve for oscillatory flow). This formula is composed of one part representing stirring of the sediment (the shear stress term) and another term describing the transport (the longshore current speed).

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